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Lecture

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CHAPTER 8 STEADY-STATE RESPONSE ANALYSES

After completing this chapter, the students will be able to:

- Find the steady-state error for unity feedback systems (type 0, 1 and 2),
- Find the steady-state error for non-unity feedback systems,
- Design the gain of a closed-loop system to meet a steady-state error specification,
- Find the steady-state error for disturbance inputs.

1. Introduction

The steady state error is a measure of system accuracy. These errors arise from the nature of the inputs, system type and from nonlinearities of system components such as static friction, backlash, etc. These are generally aggravated by amplifiers drifts, aging or deterioration. The steady state performance of a stable control system is generally judged by its steady state error to step, ramp and parabolic inputs.

Consider a unity feedback system as shown in the Fig. 1. The difference between input and output is the error signal E(s).
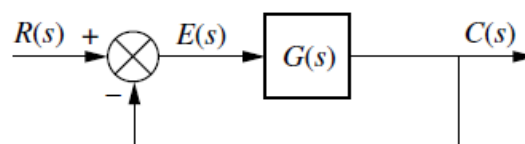

Fig.1 Closed-loop unity feedback control system

The closed loop transfer function (CLTF) is:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$
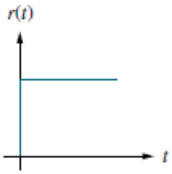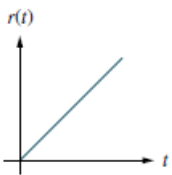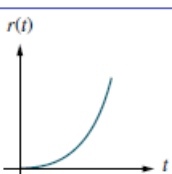
As we know $C(s) = E(s) \times G(s)$

Therefore,

$$E(s) = \frac{1}{1 + G(s)} \times R(s)$$

Steady-state error e_{ss} may be found using the Final Value Theorem (FVT) as follows:

$$e(\infty) = E_{ss} = \lim_{S \to 0} \frac{SR(s)}{1 + G(s)}$$

The above equation shows that the steady state error depends upon the input R(s) and the forward transfer function G(s). The expression for steady-state errors for various types of standard test signals shown below:

Waveform	Name	Physical interpretation	Time function	Laplace transform
$r(t)$	Step	Constant position	1	$\frac{1}{s}$
$r(t)$	Ramp	Constant velocity	t	$\frac{1}{s^2}$
$r(t)$	Parabola	Constant acceleration	$\frac{1}{2}t^2$	$\frac{1}{s^3}$

2. Steady state error and standard test input

2.1 Step input

Input is independent on time; $r(t) = 1$ OR R(s)=1/S

$$E_{ss} = \lim_{S \to 0} \frac{SR(s)}{1 + G(s)} = \lim_{S \to 0} \frac{S \times 1/S}{1 + G(s)} = \lim_{S \to 0} \frac{1}{1 + G(s)}$$

$$E_{ss} = \frac{1}{1 + \lim_{S \to 0} G(s)} = \frac{1}{1 + K_p}$$

Where K_p is the ***position error coefficient*** and equals G(0) or $K_p = \lim_{S \to 0} G(s)$

2.2 Ramp input

Input is dependent on time; $r(t) = t$ OR $R(s)=1/S^2$

$$E_{ss} = \lim_{S \to 0} \frac{SR(s)}{1 + G(s)} = \lim_{S \to 0} \frac{S \times 1/S^2}{1 + G(s)} = \lim_{S \to 0} \frac{1}{S + SG(s)}$$

$$E_{ss} = \frac{1}{\lim_{S \to 0} SG(s)} = \frac{1}{1 + K_v}$$

Where K_v is the **_velocity error Coefficient_** and equals $K_v = \lim_{S \to 0} S\, G(s)$

2.3 Parabolic input

Input is dependent on time; $r(t) = 0.5\, t^2$ OR $R(s)=1/S^3$

$$E_{ss} = \lim_{S \to 0} \frac{SR(s)}{1 + G(s)} = \lim_{S \to 0} \frac{S \times 1/S^3}{1 + G(s)} = \lim_{S \to 0} \frac{1}{S^2 + S^2 G(s)}$$

$$E_{ss} = \frac{1}{\lim_{S \to 0} S^2 G(s)} = \frac{1}{1 + K_a}$$

Where K_a is the **_acceleration error Coefficient_** and equals $K_a = \lim_{S \to 0} S^2\, G(s)$

From the above equations, it is clear that if the system input is step function, we must calculate the position error coefficient (K_p). Also, if the system input is ramp function, we must calculate the velocity error coefficient (K_v). Moreover, if the system input is parabolic function, we must calculate the acceleration error coefficient (K_a).

3. Steady state error and system type

The system type refers to the how many poles of G(s) are located at the origin. For G(S) given below, the system is of **_n-type_**.

$$G(s) = \frac{K(S + Z_1)(S + Z_2) \dots (S + Z_j)}{S^n(S + P_1)(S + P_2) \dots (S + P_k)}$$

3.1 Type zero system (n=0)

In that case, the calculation of the three coefficients K_p, K_v and K_a are

$$K_p = \lim_{S \to 0} G(s) = \frac{K(Z_1)(Z_2) \dots (Z_j)}{S^0(P_1)(P_2) \dots (P_k)} = value$$

$$K_v = \lim_{S \to 0} SG(s) = \frac{0 \times K(Z_1)(Z_2) \dots (Z_j)}{S^0(P_1)(P_2) \dots (P_k)} = 0$$

$$K_a = \lim_{S \to 0} S^2 G(s) = \frac{0 \times 0 \times K(Z_1)(Z_2) \dots (Z_j)}{S^0(P_1)(P_2) \dots (P_k)} = 0$$

The steady-state error corresponds to each coefficient is calculated as follows:

$$E_{ss} = \frac{1}{1 + K_p} = \frac{1}{1 + value} = value$$

$$E_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty$$

$$E_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

Thus, a system with n = 0, or no integration in G(s) has

• A constant position error,

• Infinite velocity error and

• Infinite acceleration error

3.2 Type one system (n=1)

In that case, the calculation of the three coefficients K_p, K_v and K_a are

$$K_p = \lim_{S \to 0} G(s) = \frac{K(Z_1)(Z_2) \dots (Z_j)}{S^1(P_1)(P_2) \dots (P_k)} = \infty$$

$$K_v = \lim_{S \to 0} SG(s) = \frac{\cancel{S} \times K(Z_1)(Z_2) \dots (Z_j)}{\cancel{S^1}(P_1)(P_2) \dots (P_k)} = value$$

$$K_a = \lim_{S \to 0} S^2 G(s) = \frac{\cancel{S} \times 0 \times K(Z_1)(Z_2) \dots (Z_j)}{\cancel{S^1}(P_1)(P_2) \dots (P_k)} = 0$$

The steady-state error corresponds to each coefficient is calculated as follows:

$$E_{ss} = \frac{1}{1 + K_p} = \frac{1}{1 + \infty} = 0$$

$$E_{ss} = \frac{1}{K_v} = \frac{1}{value} = value$$

$$E_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

Thus, a system with n =1, or with one integration in G(s) has

• A zero-position error,

• A constant velocity error and

• Infinite acceleration error

3.3 Type two system (n=2)

In that case, the calculation of the three coefficients K_p, K_v and K_a are

$$K_p = \lim_{S \to 0} G(s) = \frac{K(Z_1)(Z_2) \dots (Z_j)}{S^2(P_1)(P_2) \dots (P_k)} = \infty$$

$$K_v = \lim_{S \to 0} SG(s) = \frac{\cancel{S} \times K(Z_1)(Z_2) \dots (Z_j)}{\cancel{S^2}(P_1)(P_2) \dots (P_k)} = \infty$$

$$K_a = \lim_{S \to 0} S^2 G(s) = \frac{\cancel{S^2} \times K(Z_1)(Z_2) \dots (Z_j)}{\cancel{S^2}(P_1)(P_2) \dots (P_k)} = value$$

The steady-state error corresponds to each coefficient is calculated as follows:

$$E_{ss} = \frac{1}{1 + K_p} = \frac{1}{1 + \infty} = 0$$

$$E_{ss} = \frac{1}{K_v} = \frac{1}{\infty} = 0$$

$$E_{ss} = \frac{1}{K_a} = \frac{1}{value} = value$$

Thus, a system with n = 2, or with one integration in G(s) has

• A zero-position error,

• A zero-velocity error and

• A constant acceleration error.

Summary

	Step Input	Ramp Input	Parabolic Input
Type 0 system	$\dfrac{1}{1 + K_p}$	∞	∞
Type 1 system	*0*	$\dfrac{1}{K_v}$	∞
Type 2 system	*0*	*0*	$\dfrac{1}{K_a}$

$$K_p = \lim_{s \to 0} G(s) \qquad K_v = \lim_{s \to 0} sG(s) \qquad K_a = \lim_{s \to 0} s^2 G(s)$$
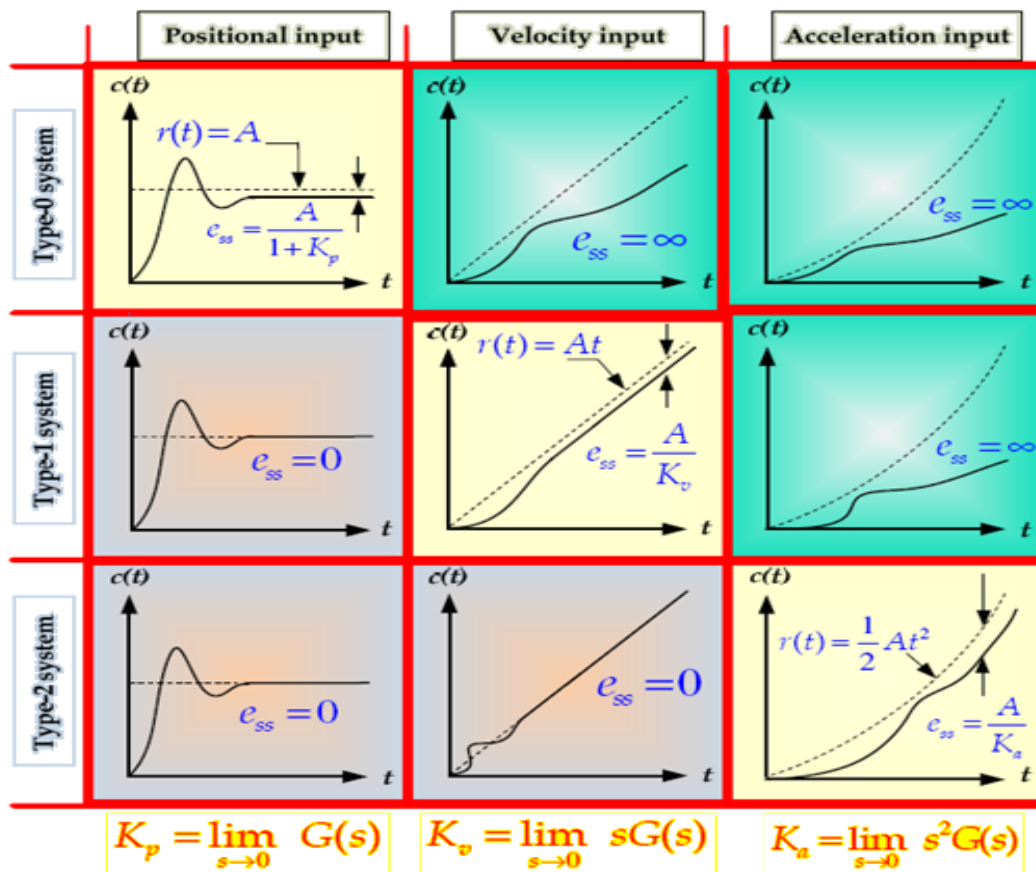
Relationships between input, system type, static error constants, and steady-state errors

Input	Steady-state error formula	Type 0 Static error constant	Error	Type 1 Static error constant	Error	Type 2 Static error constant	Error
Step, $u(t)$	$\dfrac{1}{1+K_p}$	$K_p = $ Constant	$\dfrac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\dfrac{1}{K_v}$	$K_v = 0$	∞	$K_v = $ Constant	$\dfrac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2 u(t)$	$\dfrac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a = $ Constant	$\dfrac{1}{K_a}$

Example #1

Find the steady-state errors for inputs of 5u(t), 5tu(t), and $5t^2u(t)$ to the system shown below. The function u(t) is the unit step.
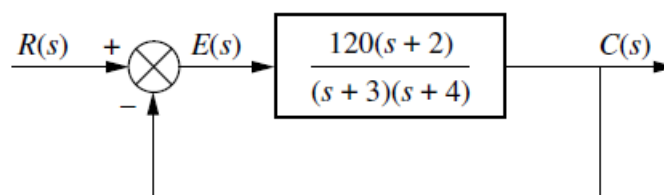
For step input 5u(t), we must calculate the position error coefficient (K_p):

$$K_p = \lim_{S \to 0} G(S) = \frac{120 \times 2}{3 \times 4} = 20$$

$$E_{ss} = \frac{5}{1 + 20} = \frac{5}{21}$$

For ramp input 5tu(t), we must calculate the velocity error coefficient (K_v):

$$K_v = \lim_{S \to 0} S\,G(S) = \frac{0 \times 120 \times 2}{3 \times 4} = 0$$

$$E_{ss} = \frac{5}{0} = \infty$$

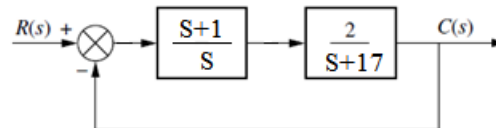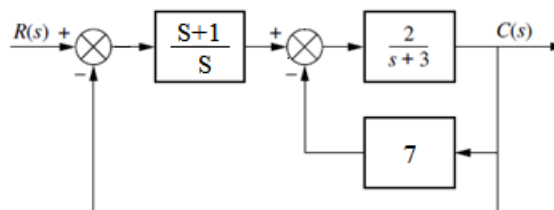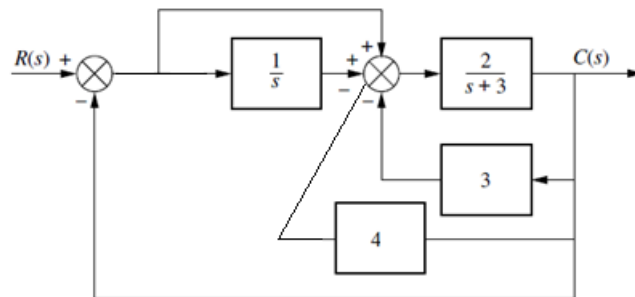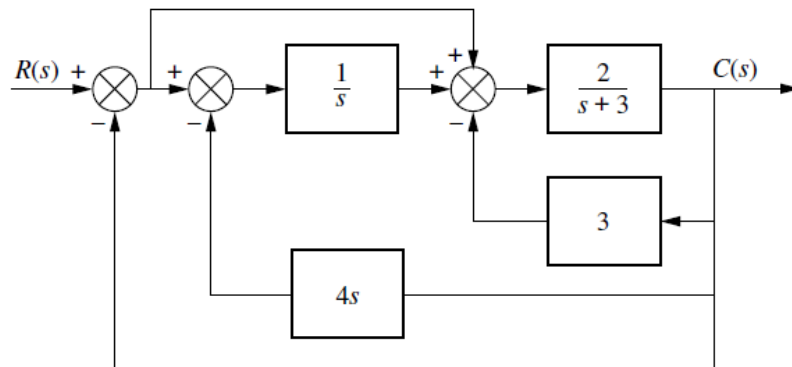For parabolic input $5t^2$ u(t), we must calculate the acceleration error coefficient (K_a):

$$K_a = \lim_{S \to 0} S^2\,G(S) = \frac{0 \times 120 \times 2}{3 \times 4} = 0$$

$$E_{ss} = \frac{2 \times 5}{0} = \infty$$

Example #2:

Find the steady-state errors for inputs of 5u(t), 5tu(t), and $5t^2$u(t) to the system shown below. The function u(t) is the unit step.
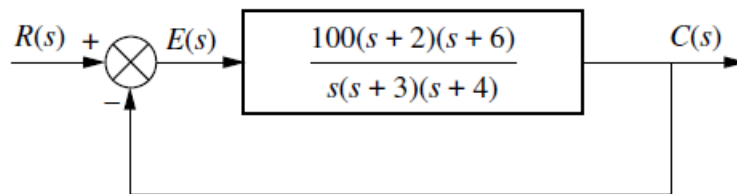
For step input 5u(t), we must calculate the position error coefficient (K_p):

$$K_p = \lim_{S \to 0} G(S) = \frac{100 \times 2 \times 6}{0 \times 3 \times 4} = \infty$$

$$E_{ss} = \frac{5}{1 + \infty} = 0$$

For ramp input 5tu(t), we must calculate the velocity error coefficient (K_v):

$$K_v = \lim_{S \to 0} S\,G(S) = \frac{100 \times 2 \times 6}{3 \times 4} = 100$$

$$E_{ss} = \frac{5}{100} = 0.05$$

For parabolic input $5t^2$ u(t), we must calculate the acceleration error coefficient (K_a):

$$K_a = \lim_{S \to 0} S^2\,G(S) = \frac{0 \times 100 \times 2 \times 6}{3 \times 4} = 0$$

$$E_{ss} = \frac{2 \times 5}{0} = \infty$$

Example #3:

Find the steady-state errors for inputs of 5u(t), 5t u(t), and $5t^2$ u(t) to the system shown in Fig. below. The function u(t) is the unit step.

$$G(s) = \frac{2(S + 1)}{S(S + 17)}$$

For step input 5u(t), we must calculate the position error coefficient (K_p):

$$K_p = \lim_{S \to 0} G(S) = \frac{2 \times 1}{0 \times 17} = \infty$$

$$E_{ss} = \frac{5}{1 + \infty} = 0$$

For ramp input 5tu(t), we must calculate the velocity error coefficient (K_v):

$$K_v = \lim_{S \to 0} S \, G(S) = \frac{2 \times 1}{17} = \frac{2}{17}$$

$$E_{ss} = \frac{5 \times 17}{2} = 42.5$$

For parabolic input $5t^2$ u(t), we must calculate the acceleration error coefficient (K_a):

$$K_a = \lim_{S \to 0} S^2 \, G(S) = \frac{0 \times 2 \times 1}{17} = 0$$

$$E_{ss} = \frac{2 \times 5}{0} = \infty$$

Example #4

For the system shown below, find K so that there is 10% error in the steady state
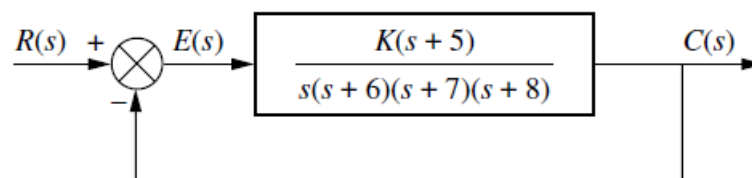
Since the system type is 1, the error stated in the problem must apply to a ***ramp input***; only a ramp yields a finite error for in a type 1 system. Thus,

$$K_v = \lim_{S \to 0} S \, G(S) = \frac{K \times 5}{6 \times 7 \times 8} = \frac{5K}{336}$$

$$E_{ss} = \frac{336}{5K} = 0.1$$

$$K = 672$$

4. Steady state error of non-unity feedback systems

Consider the non-unity feedback control system shown in Fig. 2.
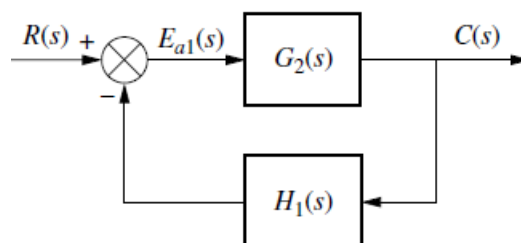
Fig. 2, non-unity feedback system

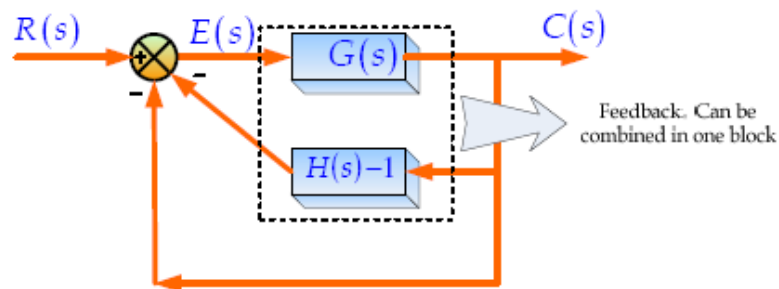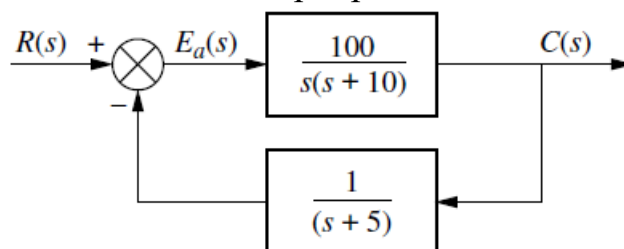Add to the block shown in Fig. 2, two feedback blocks H(S) = +1 and H(S) = −1 as shown in Fig. 3.
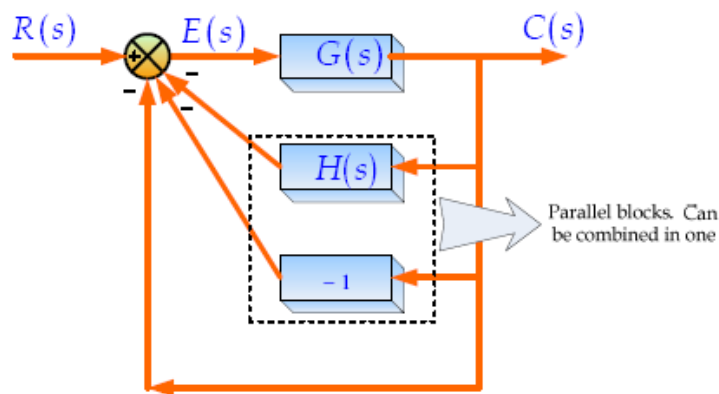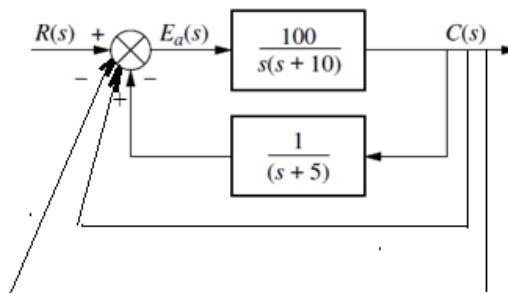
Fig.3
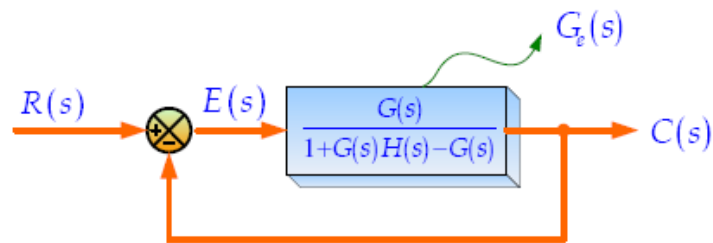
Example #5

For the system shown below, find

• The system type,
• Appropriate error constant associated with the system type, and
• The steady state error for unit step input

Add to the block shown above, two feedback blocks $H(S) = +1$ and $H(S) = -1$ as shown below.

The equivalent feed-forward transfer function is

$$\frac{100(S+5)}{S^3+15S^2-50S-400}$$

System Type: 0

Appropriate error constant is K_p

$$K_p = \lim_{S\to0} G(S) = \frac{100\times(0+5)}{0^3+15\times0^2-50\times0-400} = -\frac{500}{400} = -1.25$$

Steady State Error e_{ss}

$$E_{ss} = \frac{1}{1+K_p} = \frac{1}{1-1.25} = -4$$

The negative value for E_{ss} implies that the output is larger than the input step.

Example #6

An engine speed control system is shown below.

1) Calculate E_{ss} for step input with magnitude A when $K_2 = 0$

2) Calculate E_{ss} for step input with magnitude A when $K_2 \neq 0$

3) Calculate E_{ss} for ramp input with slope A when $K_2 = 0$

4) Calculate E_{ss} for ramp input with slope A when $K_2 \neq 0$

5) Given $K_1=1.2$, $K_2=8.4$, and $T=0.5$, what value of K gives $K_v=6$ for unit ramp input. Find the corresponding steady-state error. Sketch the input and output as functions of time in that case.
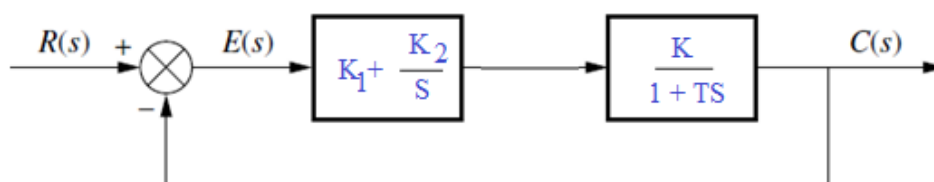
$$G(s) = \frac{K(K_1 S + K_2)}{S(1 + TS)}$$

1) When $K_2 = 0$, the above G(s) is reduced to:

$$G(s) = \frac{KK_1}{1 + TS}$$

$$K_p = \lim_{S \to 0} G(S) = KK_1$$

$$E_{ss} = \frac{1}{1 + K_p} = \frac{1}{1 + KK_1}$$

For a step input with magnitude A;

$$E_{ss} = \frac{A}{1 + KK_1}$$

2) When $K_2 \neq 0$, the open-loop T.F reverts to the original form:

$$G(s) = \frac{K(K_1 S + K_2)}{S(1 + TS)}$$

Which represent a type 1 system.

$$K_p = \lim_{S \to 0} G(S) = \infty$$

$$E_{ss} = \frac{1}{1 + K_p} = \frac{1}{1 + \infty} = 0$$

3) When $K_2 = 0$ and the input is ramp;

$$K_v = \lim_{S \to 0} S\, G(S) = 0$$

$$e_{ss} = \frac{A}{K_v} = \infty$$

When $K_2 \neq 0$,

$$K_v = \lim_{S \to 0} S\, G(S) = KK_2$$

$$e_{ss}(t) = \frac{A}{K_v} = \frac{A}{KK_2}$$

4. Given $K_1 = 1.2$, $K_2 = 8.4$, and $T = 0.5$, it is required that
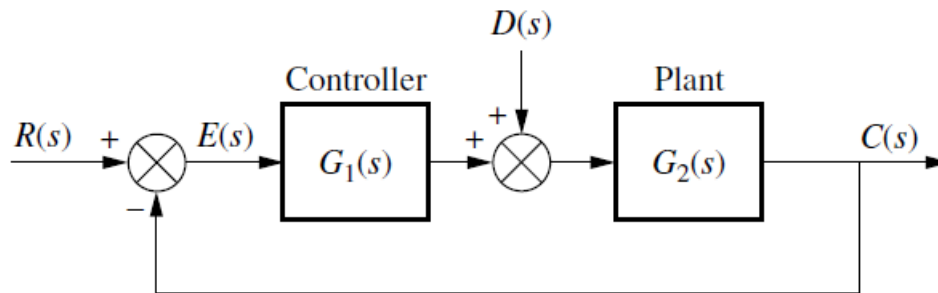
$$K_v = 6 = 8.4K$$

Hence the result

$$K = 0.714$$

The steady-state error of the closed-loop system becomes

$$e_{ss}(t) = \frac{1}{K_v} = 0.167$$

5. Steady-state Error for Disturbances:

Using super position to calculate the system T.F.

When D(s)=0

$$\frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)}$$

When R(s)=0

$$\frac{C(s)}{D(s)} = \frac{G_2(s)}{1 + G_1(s)G_2(s)}$$

$$C(s) = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)} R(s) + \frac{G_2(s)}{1 + G_1(s)G_2(s)} D(s)$$

But the error signal is given by

$$E(s) = R(s) - C(s)$$

$$E(s) = R(s) - \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)} R(s) - \frac{G_2(s)}{1 + G_1(s)G_2(s)} D(s)$$

$$E(s) = \left(1 - \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)}\right) R(s) - \frac{G_2(s)}{1 + G_1(s)G_2(s)} D(s)$$

$$E_{ss} = \lim_{s \to 0} S \left(1 - \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)}\right) R(s) - \lim_{s \to 0} S \left(\frac{G_2(s)}{1 + G_1(s)G_2(s)}\right) D(s)$$

The first term, is the steady-state error due to R(s). The second term, is the steady-state error due to the disturbance.

Example #7

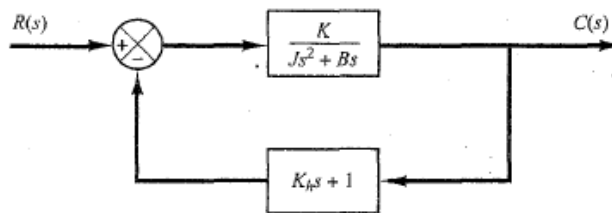Consider the servo system with tachometer feedback shown in Figure.

Obtain the error signal $E(s)$ when both the reference input $R(s)$ and disturbance input $D(s)$ are present. Obtain also the steady-state error when the system is subjected to a reference input (unit-ramp input) and disturbance input (step input of magnitude d).
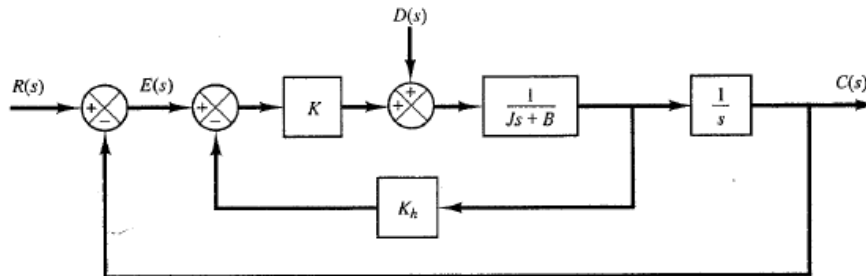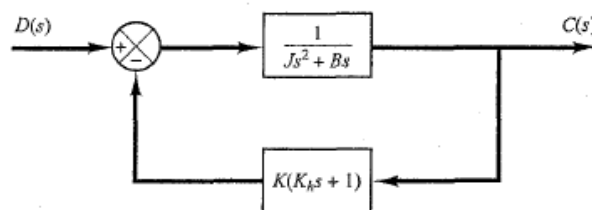

Using super position:

(when D(S) = 0)

$$\frac{C(s)}{R(s)} = \frac{K}{JS^2 + (B + KK_h)S + K}$$

(when R(S) = 0)

$$\frac{C(s)}{R(s)} = \frac{1}{JS^2 + (B + KK_h)S + K}$$

The overall output is

$$C(s) = \frac{1}{JS^2 + (B + KK_h)S + K}[K \times R(s) + D(s)]$$

Since E(s) = R(s) – C(s);

$$E(s) = R(s) - \frac{KR(s) + D(s)}{JS^2 + (B + KK_h)S + K}$$

$$E(s) = \frac{1}{JS^2 + (B + KK_h)S + K}\{[JS^2 + (B + KK_h)S]R(s) - D(s)\}$$

Hence the steady-state error can be obtained as follows:

$$\lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$$

$$= \lim_{s \to 0} \frac{s}{Js^2 + (B + KK_h)s + K}\left[s(Js + B + KK_h)R(s) - D(s)\right]$$

$$= \lim_{s \to 0}\left[\frac{B + KK_h}{K} s^2 R(s) - \frac{1}{K} sD(s)\right]$$

Since $R(s) = 1/s^2$ (unit-ramp input) and $D(s) = d/s$ (step input of magnitude d) the steady-state error is

$$\lim_{t \to \infty} e(t) = \lim_{s \to 0}\left[\frac{B + KK_h}{K}\frac{s^2}{s^2} - \frac{1}{K}\frac{sd}{s}\right]$$

$$= \frac{B + KK_h}{K} - \frac{d}{K}$$

Example #8
For the control system shown below in Fig A,
    a) Determine the values of gain K and the time constant T so that the system response for unit-step input is as shown in Fig. B.
    b) With these values of K and $T,$ obtain in part (a), find the rise time and peak time and percentage overshoot.
    c) Calculate the position error coefficient and the steady-state error.

Fig. A
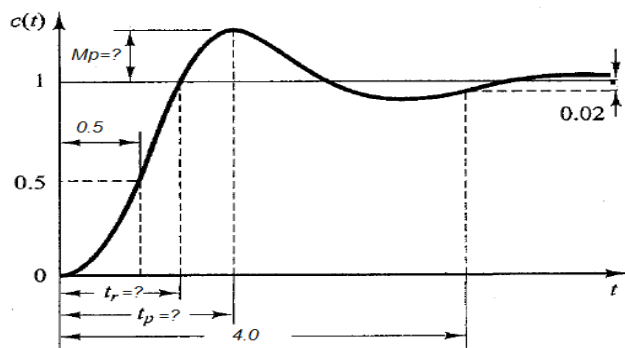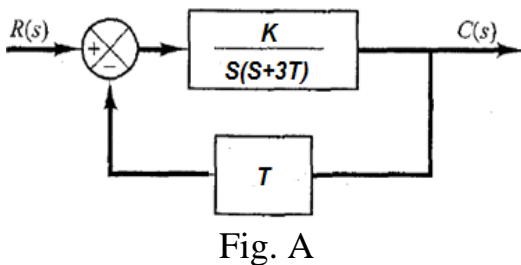
Fig. B

From system response (Fig. B),
$T_S = 4$ (based on 2% tolerance) and $T_d = 0.5$
since

$$T_S = \frac{4}{\xi\, \omega_n} = 4$$

Therefore,

$$\xi \, \omega_n = 1$$

Also, we have the delay time $T_d = 0.5$

$$T_d = \frac{1 + 0.7\xi}{\omega_n} = 0.5$$

Multiply both sides by ξ, then

$$T_d = \frac{\xi + 0.7\xi^2}{\xi \, \omega_n} = 0.5$$

But we get that, $\xi \, \omega_n = 1$, therefore
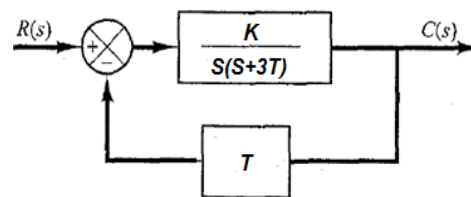
$$0.7\xi^2 + \xi - 0.5 = 0$$

Solving this equation

$$\xi = 0.392281 \; (accepted)$$
$$\xi = -1.82085 \; (rejected)$$
$$\omega_n = \frac{1}{\xi} = 2.5492 \; rad/s$$

From the system block diagram

The system T.F.

$$\frac{C(s)}{R(s)} = \frac{K}{S^2 + 3Ts + KT}$$

The system characteristic equation is

$$S^2 + 3Ts + KT = 0$$

The standard form of 2nd order system characteristic equation is

$$S^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

By comparing

$$2\xi\omega_n = 3T$$
$$T = \frac{2}{3}$$

Also,

$$\omega_n^2 = KT$$
$$K = \frac{\omega_n^2}{T} = 9.7476$$

The % maximum over shoot M_p

$$M_p = e^{-\frac{\pi \zeta}{\sqrt{1 - \zeta^2}}} \times 100 = 26.19 \%$$

Rise Time:

$$T_r = \frac{\pi - \beta}{\omega_n\sqrt{1 - \xi^2}}$$

B = cos^{-1}(ξ) = 66.90349° = 1.167686 rad

$$T_r = \frac{\pi - 1.167686}{2.5492\sqrt{1 - 0.392281^2}} = 0.842 \; sec.$$

Peak Time:
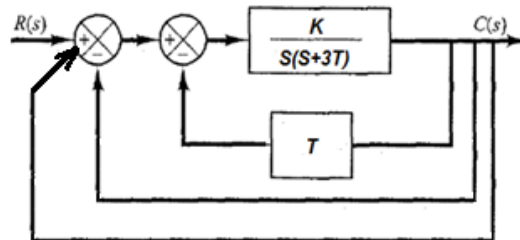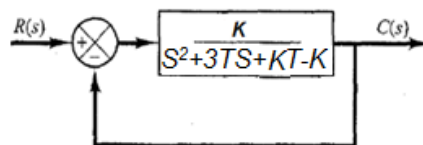
$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$T_p = \frac{\pi}{2.5492\sqrt{1-0.392281^2}} = 1.34\ sec.$$

To get the steady-state error and position error coefficient, the system must be unity feedback, so we will add +ve and –ve feedback as shown in fig


Then the unity feedback system will be

$$K_p = \lim_{S\to 0} G(s) = \frac{K}{KT-K} = \frac{1}{T-1} = \frac{1}{0.6667-1} = -3$$

$$E_{ss} = \frac{1}{1+K_p} = \frac{1}{1-3} = -0.5$$

Example #9:

The control system, shown in Fig. below, is subjected to a unit ramp function,

    a) Determine the value of K and K_h such that the system has an overshoot of 16.303% and a damped natural frequency of 3.4641 rad/sec.

    b) Calculate the rise time, peak time and settling time based on ±2% tolerance

    c) Define the system type

    d) Calculate the steady-state error of that system

    e) If an integrator is added in the forward path after the 1st summing point directly, calculate the new steady-state error.
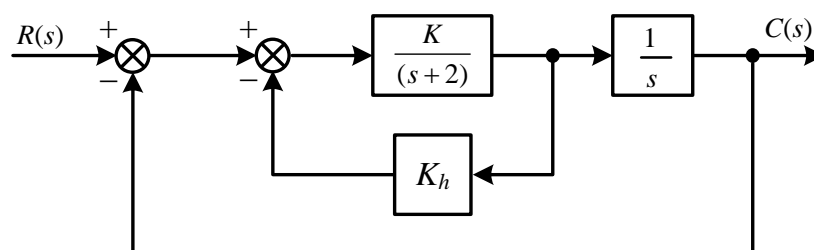
Maximum overshoot $= 0.16303 = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$ taking (ln) for both sides

$$-1.813821 = -\frac{\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$3.29 = \frac{\pi^2\zeta^2}{1-\zeta^2}$$
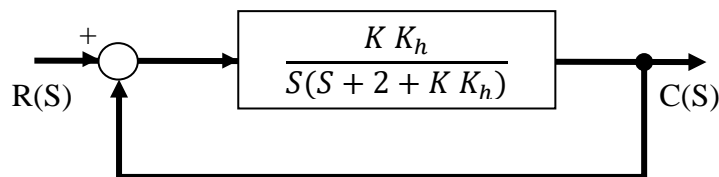
$$0.333347 - 0.33347\,\zeta^2 = \zeta^2$$

$$\zeta^2 = \frac{0.333347}{1.333347} = 0.25$$

$$\zeta = \sqrt{0.25} = 0.5$$

Since the damped natural frequency $(\omega_d) = \omega_n \sqrt{1-\zeta^2}$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{3.4641}{\sqrt{1-0.25}} = 4\ rad/sec.$$

The block diagram can be simplified as follows

The closed loop transfer function $\dfrac{C(S)}{R(S)} = \dfrac{K}{S^2 + (2+KK_h)S + K}$

But the general form T.F. of the 2nd order system is $\dfrac{C(S)}{R(S)} = \dfrac{\omega_n{}^2}{S^2 + (2\zeta\omega_n)S + \omega_n{}^2}$

By comparing the coefficient of both T.F.

$$K = \omega_n{}^2 = 16 \qquad \#\#$$

$$2 + KK_h = 2\zeta\omega_n$$

$$K_h = 0.125 \qquad \#\#$$

The angle $\beta = cos^{-1}(0.5) * \frac{\pi}{180} = 1.0472\ rad$

Rise time $(T_r) = \dfrac{\pi - \beta}{\omega_d} = \dfrac{\pi - 1.0472}{3.4641} = 0.605\ sec.$ ##

Peak time $(T_p) = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{3.4641} = 0.907\ sec.$ ##

Settling time (T_s) based on $\pm2\%$ tolerance $= \dfrac{4}{\zeta\omega_n} = \dfrac{4}{0.5*4} = 2.0\ sec.$ ##

The system type = 1

The velocity error coefficient $K_V = \lim\limits_{S\to0} S\,G(s)H(s) = \lim\limits_{S\to0} S\,\dfrac{K}{S(S+2+K\,K_h)} = \dfrac{K}{2+K\,K_h} =$

$\dfrac{16}{2+2} = 4$

The steady-state error $(E_{ss}) = \dfrac{1}{K_V} = 0.25$ ##

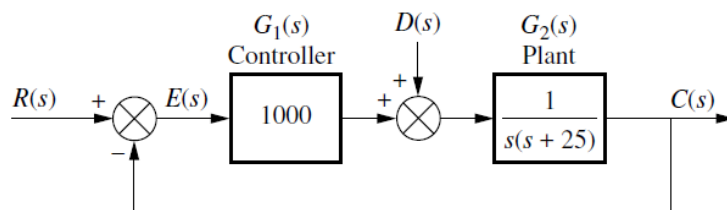When an integrator is added in the forward path after the 1st summing point directly as shown in Fig.
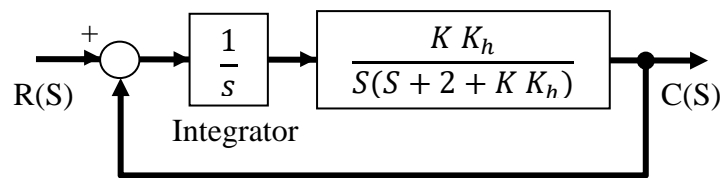
The new $G(s)H(s) = \dfrac{K\,K_h}{S^2\,(S+2+K\,K_h)}$

The new velocity error coefficient $K_V = \lim_{S\to 0} S\,G(s)H(s) = \lim_{S\to 0} S\,\dfrac{K}{S^2(S+2+K\,K_h)} = \dfrac{K}{0} = \infty$

The new steady-state error $(E_{ss}) = \dfrac{1}{K_V} = \dfrac{1}{\infty} = 0$ ##

Example #10:

For the system given below, find the steady-state error due to step input for both of reference input R(s) and the disturbance D(s).

If D(s)=0

$$\frac{C(s)}{R(s)} = \frac{1000}{S(S+25)+1000}$$

If R(s)=0

$$\frac{C(s)}{D(s)} = \frac{1}{S(S+25)+1000}$$

$$C(s) = \frac{1000}{S(S+25)+1000}\,R(s) + \frac{1}{S(S+25)+1000}\,D(s)$$

$$E(s) = \left(1 - \frac{1000}{S(S+25)+1000}\right)R(s) - \frac{1}{S(S+25)+1000}\,D(s)$$

Replace the values of R(s) and D(s) by 1/S

$$E(s) = \left(1 - \frac{1000}{S(S+25)+1000}\right)\frac{1}{s} - \frac{1}{S(S+25)+1000}\,\frac{1}{s}$$

$$SE(s) = \left(1 - \frac{1000}{S(S+25)+1000}\right) - \frac{1}{S(S+25)+1000}$$

$$E_{ss} = \lim_{s\to 0} SE(s) = 1 - \frac{1000}{1000} - \frac{1}{1000} = -0.001$$

Sheet 6 *(Steady-State Analysis)*

1) For unity feedback control systems given below, find the position (K_p), velocity (K_v) and acceleration (K_a) error coefficients.

$$G(s) = \frac{10}{(s+1)(2s+1)}$$
(c) $$G(s) = \frac{K}{s^2(0.5s+1)(s+1)}$$

$$G(s) = \frac{K}{s(s+1)(2s+1)}$$
(d) $$G(s) = \frac{K(s+4)}{s^2(s^2+6s+2)}$$

2) For the system shown in Fig. 1, design the value of K so that there is 1% error in the steady state for an input 100tu(t).
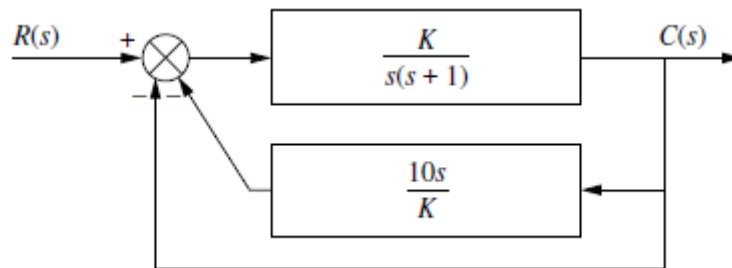

Fig. 1

3) For the system shown in Fig. 2, find

   • The system type
   • Appropriate error constant associated with the system type, and find the corresponding steady-state error.
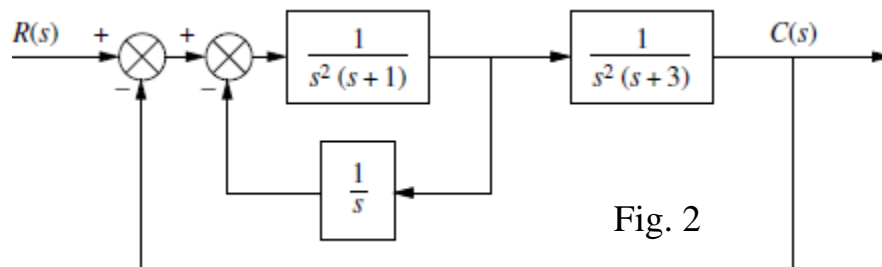
Fig. 2

4) Consider the system shown in Fig. 3. Prove that the steady-state error for a unit ramp input is $\frac{2\zeta}{\omega_n}$. Also show that the damping ratio is $\frac{B}{2\sqrt{KJ}}$ and the undamped natural frequency is $\sqrt{\frac{K}{J}(1-\zeta^2)}$
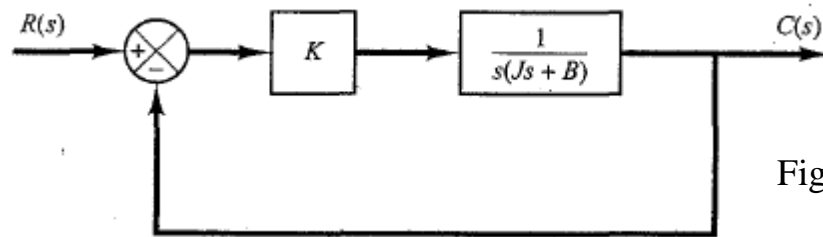
Fig. 3

5) Consider the system shown in Fig. 4(a). The steady-state error to a unit-ramp input is $e_{ss} = \dfrac{2\zeta}{\omega_n}$. Show that the steady-state error for following a ramp input may be eliminated if the input is introduced to the system through a proportional-plus-derivative filter, as shown in Fig. 4(b), and the value of k is properly set.
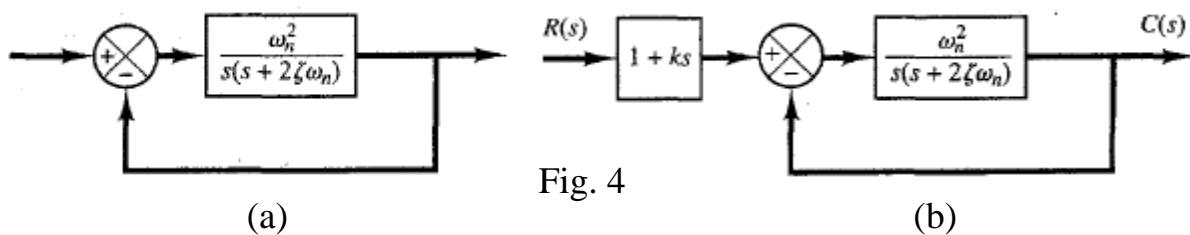

Fig. 4

(a) (b)

6) Consider servo system with tachometer feedback shown in Fig. 5. Obtain the error signal $E(s)$ when both the reference input $R(s)$ and disturbance input $D(s)$ are present. Obtain also the steady-state error when the system is subjected to a reference input (unit-ramp input) and disturbance input (step input of magnitude d).
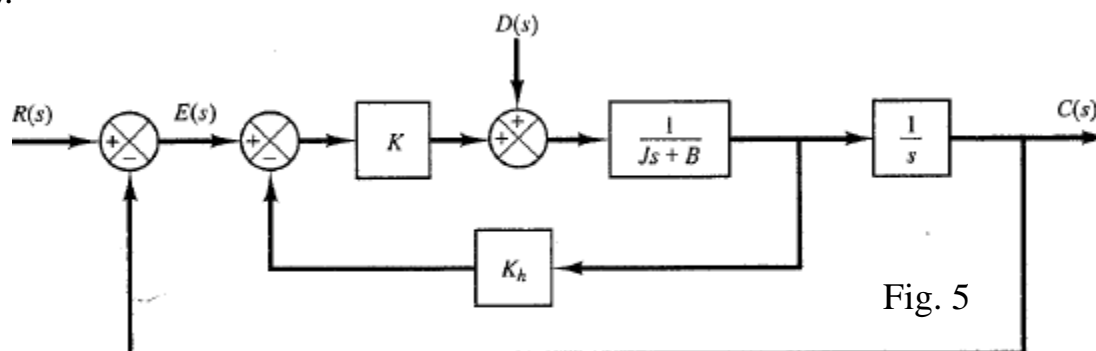
Fig. 5

7) For the control system shown in Fig. 6, Determine the steady-state error for a unit step when $K = 0.4$ and $G_p(s) = 1$. Select an appropriate value for $G_p(s)$ so that the steady-state error is equal to zero for the unit step input.
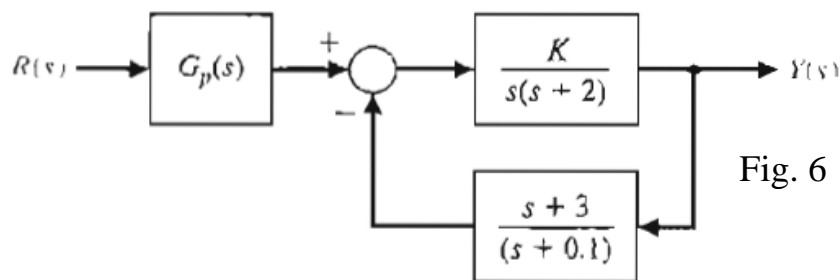
Fig. 6

8) Consider the closed-loop system in Fig. 7. Determine values of the parameters k and a so that the following specifications are satisfied:
(a) The steady-state error to a unit step input is zero.
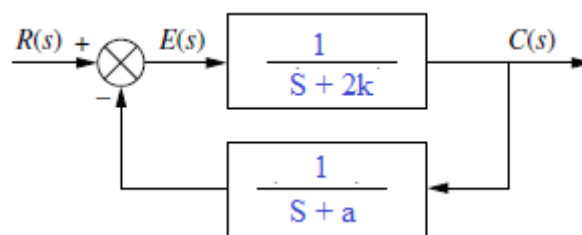(b) The closed-loop system has a percent overshoot of less than 5%.

Fig. 7

9. For the control system shown in Fig. 8, the system is subjected to an input r(t)=2t. where K_1 and K_2 are constants.
(a) Obtain the system type,
(b) Calculate the value of K_2 if the steady-state error $E_{SS} = 0.085$ and $K_1 = 3.0$,
(c) If K_1 is set to zero and K_2 is reduced to its half value obtained in (b), calculate the steady-state error (E_{SS}).
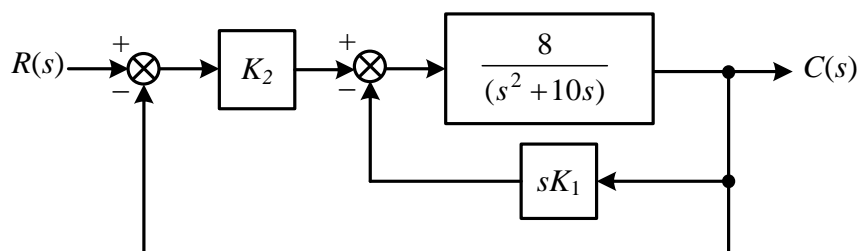

Fig. 8

10. For the system given in Fig. 9, find the steady-state error due to step input for both of reference input R(s) and the disturbance D(s).
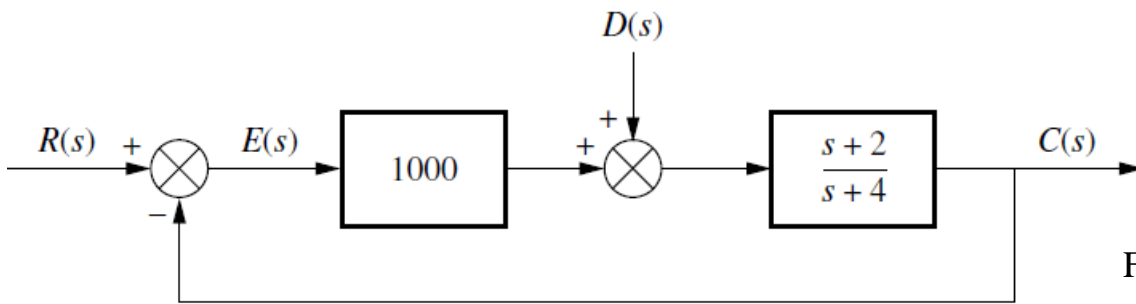
Fig. 9

11. For the system shown in Fig. 10, and Fig. 11
a) calculate K_p, K_v and K_a
b) state the system type,
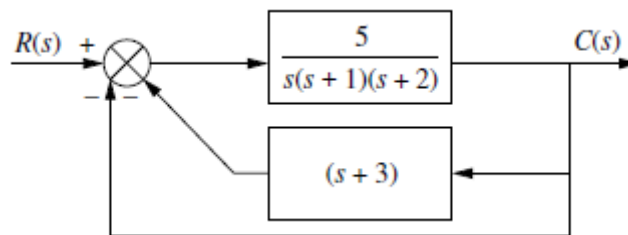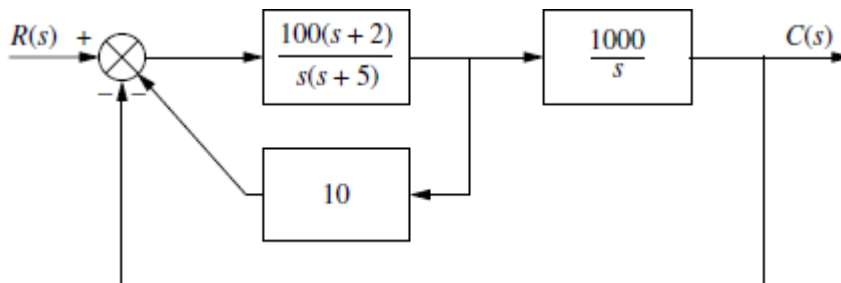c) calculate the steady-state error for input 50 t u(t)

Fig. 10

Fig. 11

12. For the system shown in Fig. 12, find the value of gain K to give a steady-state error of 0.061 at input $10t^2u(t)$
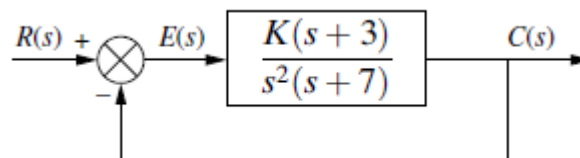

Fig. 12

13. for the unity feedback control system shown in Fig. 13, design the values of K, α, and β to meet the following requirements:

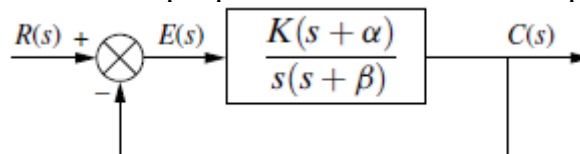The steady-state error for a unit ramp input is 0.1 and the closed-loop poles located at $-1 \pm J1$

Fig. 13

14. For the unity feedback control system shown in Fig. 14, design the values of K, α, and β to meet the following requirements:

The steady-state error for a unit step input = 0.1, damping ratio = 0.5 and the undamped natural frequency = $\sqrt{10}$
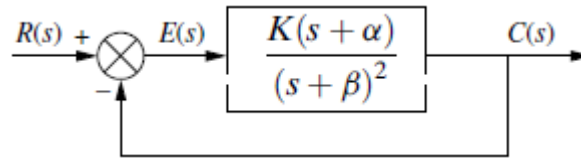

Fig. 14

15. Design the values of K_1 and K_2 in the system given in Fig. 15 to meet the following specifications: Steady-state error component due to a unit step disturbance is -0.000012; steady-state error component due to a unit ramp input is 0.003
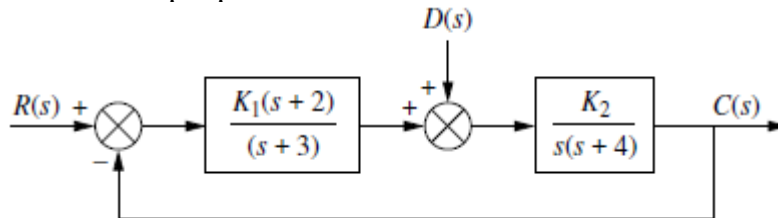

Fig. 15

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[1] Bosch, R. GmbH. Automotive Electrics and Automotive Electronics, 5th ed. John Wiley & Sons Ltd., UK, 2007.

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